Design And Analysis of Experiment

Advanced Topics in Systems Performance Evaluation

Professors: Paulo Maciel/Ricardo Massa

Group 1

Eric Borba erb@cin.ufpe.br
Erico Guedes eacg@cin.ufpe.br
Jonas Pontes jcnp@cin.ufpe.br

2015

Centro de Informática U·F·P·E
Universidade Federal de Pernambuco

Summary

- Preliminaries
  - Definition
  - Terminology
- Types of Experiment Design
  - One Factor Experiments
  - Two Factor Factorial Experiments
  - 2^k Factorial Designs
  - 2^kr Factorial Designs
  - 2^(k-p) Fractional Factorial Designs

Preliminaries

The first ninety percent of the task takes ten percent of the time, and the last ten percent takes the other ninety percent.

—Ninety-ninety rule of project schedules

Preliminaries

"Almost all science successes were preceded by failures."

Preliminaries

Experience

x

Experiment

Preliminaries

- **Thomas Edison**
  - **More than 1150 failed experiments before functional lamp**

Preliminaries

- Observing a system or process while it is in operation is an important part of the learning process, and is an integral part of understanding and learning about how systems and processes work.

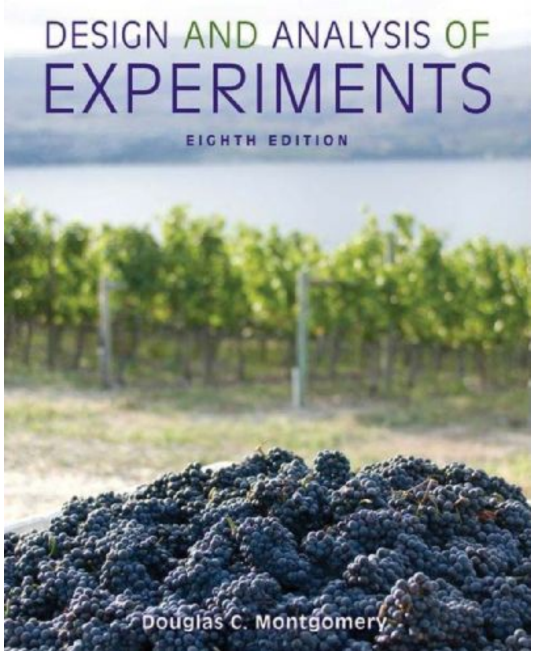
Preliminaries

- **The goal of a proper experimental design is to obtain the maximum information with the minimum number of the experiments.**

Preliminaries

- **Understand cause-and-effect**
  - **"Throughout the book I have stressed the importance of experimental design as a tool for engineers and scientists to use for *product design and development* as well as *process development and improvement*."**

Here we are!



Preliminaries

- **Understand cause-and-effect**
  - **Scientific Knowledge:**
    - **Aiming to connect each effect to one cause in order to obtain some possibility to do safe predictions related to future events.**

Preliminaries

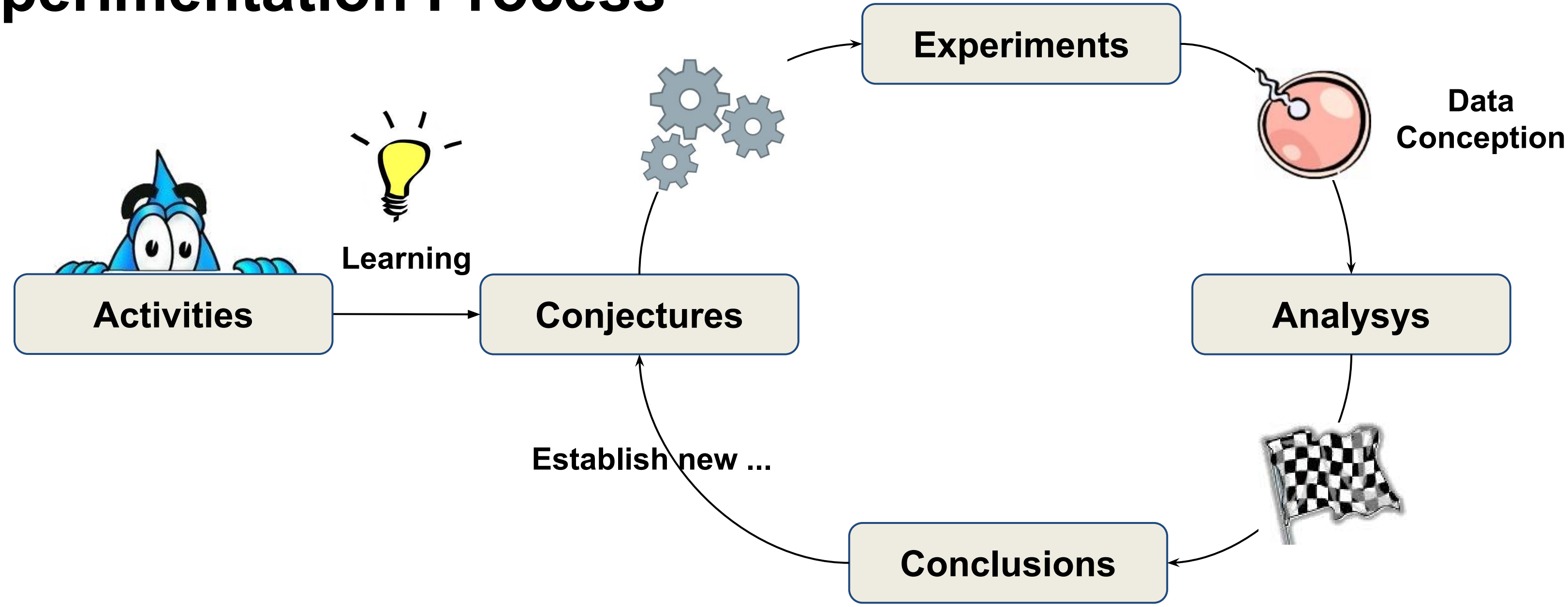
- **Definition: Experiment Design**
  - **It is a systematic method to determine the relationship between factors affecting a process and the output of that process.**
    - **it is used to find cause-and-effect relationships**
    - **this information is needed to manage process inputs in order to *optimize* the output**

Preliminaries

- What is an experiment?
  - a test or series of runs in which purposeful changes are made to the input variables of a process or system so that we may observe and identify the reasons for changes that may be observed in the output response.

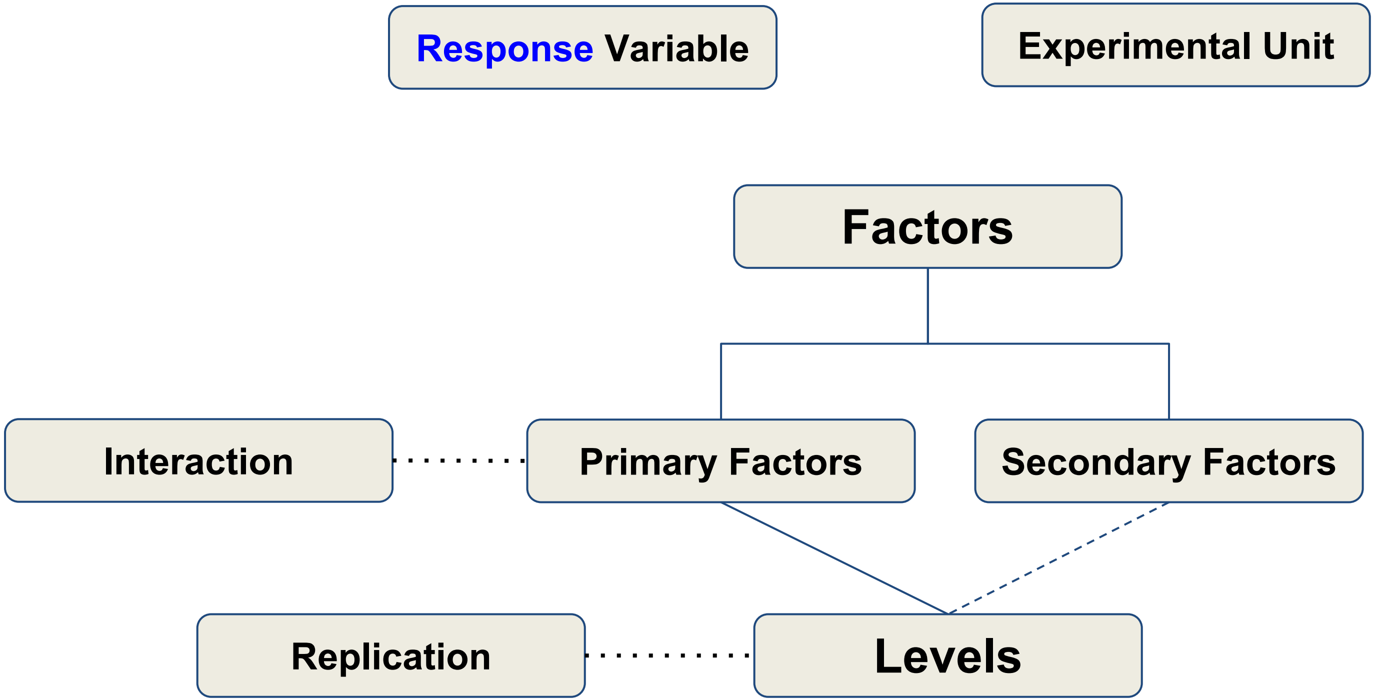
Preliminaries

- **Experimentation Process**



Preliminaries

- **Design of Experiment Terminology**



Interaction: Example

TABLE 16.1 Noninteracting Factors

	A_1	A_2
B_1	3	5
B_2	6	8

TABLE 16.2 Interacting Factors

	A_1	A_2
B_1	3	5
B_2	6	9

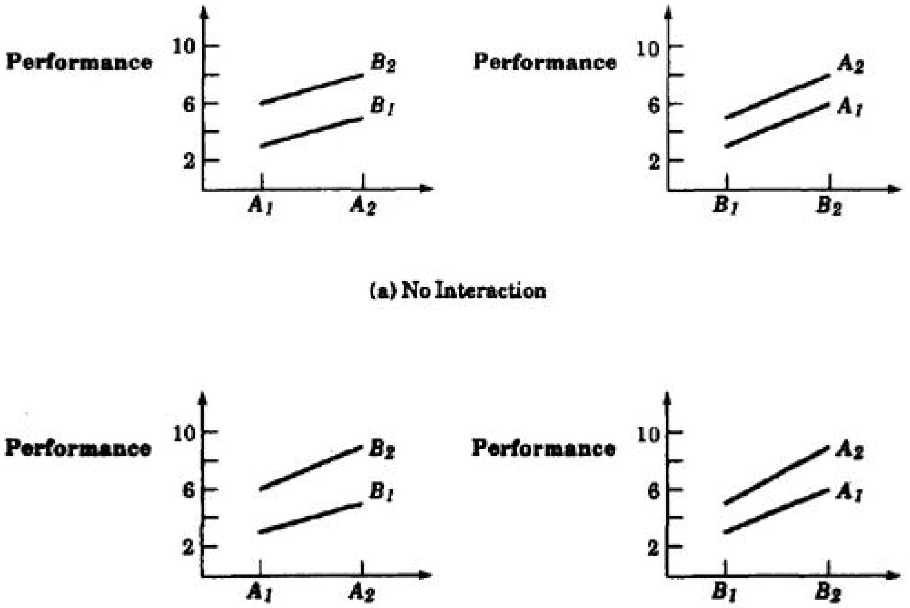


FIGURE 16.1 Graphical presentation of interacting and noninteracting factors.

Preliminaries

- **Guidelines for Design Experiments(1/3)**

1. *Recognition of* and *statement of* the problem

Reasons for running experiments:
a. Factor screening or characterization
b. Optimization
c. Confirmation
d. Discovery
e. Robustness

2. Selection of response variable

μ

σ

Performed simultaneously or in reverse order

3. Choice of factors, levels and range

- Together, steps 1, 2, and 3 are called **pre-experimental planning**

Preliminaries

- **Guidelines for Design Experiments(2/3)**

4. Choice of experimental design

Sample size (number of replications)
Suitable run order for the experimental trials

Thinking about and selecting **a tentative empirical model** to describe the results

What is the model?
- a quantitative relationship (equation) between the **response** and the important design **factors**
- low order polynomial models may be appropriate

i. First-order model:
$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \varepsilon$

ii. First-order model with interaction term:
$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_{12} x_1 x_2 + \varepsilon$

iii. Second-order model:
$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_{12} x_1 x_2 + \beta_{11} x_1^2 + \beta_{22} x_2^2 + \varepsilon$

Preliminaries

- **Guidelines for Design Experiments(3/3)**

5. Performing the experiment

Monitor the process carefully

Suggestion: **few trial runs** or **pilot runs**

6. Statistical analysis of the data

Graphical methods play an important role

hypothesis testing and **confidence interval estimation procedures** are very useful

7. Conclusions and recommendations

Follow-up runs and **confirmation** testing should also be performed to validate the conclusions from the experiment

Summary

- Preliminaries
  - Definition
  - Terminology
- Types of Experiment Design
  - One Factor Experiments
  - Two Factor Factorial Experiments
  - 2^k Factorial Designs
  - 2^kr Factorial Designs
  - 2^(k-p) Fractional Factorial Designs

Types of Experimental Designs

- Simple Design
- Full Factorial Design
- Fractional Factorial Design

Types of Experimental Designs

Simple Designs

- **One Factor at a Time (OFAT)**
  - The major disadvantage of the OFAT strategy is that it fails to consider any possible interaction between the factors
  - If the factors have interaction, this design may lead to wrong conclusions.
- Number of experiments(n) for a simple design:

$$n = 1 + \sum_{i=1}^{k} (n_i - 1)$$

Types of Experimental Designs

Simple Designs

- **One Factor at a Time (OFAT)**

The problem is to design a personal workstation, where several choices have to be made. First, a microprocessor has to be chosen for the CPU. The alternatives are the 68000, Z80, or 8086 microprocessor. Second, a memory size of 512 kbytes, 2 Mbytes, or 8 Mbytes has to be chosen. Third, the workstation could have one, two, three, or four disk drives. Fourth, the workload on the workstations could be one of three types—secretarial, managerial, or scientific. Performance also depends on user characteristics, such as whether users are at a high school, college, or postgraduate level.

$$n = 1 + \sum_{i=1}^{k} (n_i - 1)$$

Types of Experimental Designs

Full Factorial Design: 2^k

- It uses every possible combination at all levels of all factors.
- Experiments quantity:

$$n = \prod_{i=1}^{k} n_i$$

- Every possible combination of configuration is examined (factors and interactions).
- Issues: cost and time.
- 3 ways to reduce the number of experiments: levels, factors or fractional factorial designs.

Types of Experimental Designs

Fractional Factorial Designs(2^{k-p})

- **Sometimes the number of experiments is too large.**
- **In such cases, it is possible to use only a fraction of the factorial**

Example 16.1 Consider only four of the five factors in the workstation study. Let us ignore the number of disk drives for this example. We have four factors, each at three levels. Therefore, the number of experiments required is

$$\begin{aligned} n &= \text{(3 CPUs)(3 memory levels)(3 workloads)(3 educational levels)} \\ &= \text{81 experiments} \end{aligned}$$

TABLE 16.3 A Sample Fractional Factorial Design

Experiment Number	CPU	Memory Level	Workload Type	Educational Level
1	68000	512K	Managerial	High school
2	68000	2M	Scientific	Postgraduate
3	68000	8M	Secretarial	College
4	Z80	512K	Scientific	College
5	Z9D	2M	Secretarial	High school
6	Z80	8M	Managerial	Postgraduate
7	8086	512K	Secretarial	Postgraduate
8	8086	2M	Managerial	College
9	8086	8M	Scientific	High school

One Factor Experiments

Types of Variables:
- Categorical (qualitative)
- Numerical (quantitative)

- **Features**
  - Used to compare several alternatives of a single categorical variable.
  - There is no number of levels that the factor can take.
    - Unlike the 2^k designs, the number of levels can be more than 2
  - Model used

$$y_{ij} = \mu + \alpha_j + e_{ij}$$

  - Observation, mean response, effect, error term.

$$\sum_{i=1}^{r}\sum_{j=1}^{a} y_{ij} = ar\mu + r\sum_{j=1}^{a}\alpha_j + \sum_{i=1}^{r}\sum_{j=1}^{a} e_{ij}$$

One Factor Experiments

- **Model parameter:**

$$\mu = \frac{1}{ar}\sum_{i=1}^{r}\sum_{j=1}^{a} y_{ij}$$

- **Column mean:**

$$\bar{y}_{.j} = \frac{1}{r}\sum_{i=1}^{r} y_{ij}$$

- **Column effect:**

$$\alpha_j = \bar{y}_{.j} - \mu = \bar{y}_{.j} - \bar{y}_{..}$$

One Factor Experiments

- **Model parameter:**

$$\mu = \frac{1}{ar} \sum_{i=1}^{r} \sum_{j=1}^{a} y_{ij}$$

- **Column mean:**

$$\bar{y}_{.j} = \frac{1}{r} \sum_{i=1}^{r} y_{ij}$$

- **Column effect:**

$$\alpha_j = \bar{y}_{.j} - \mu = \bar{y}_{.j} - \bar{y}_{..}$$

One Factor Experiments

Example

TABLE 20.1 Analysis of the Code Size Comparison Study

R	V	Z
144	101	130
120	144	180
176	211	141
288	288	374
144	72	302

One Factor Experiments

Estimating Experimental Errors

- **The difference between the measured and the estimated response represents experimental error.**

$$\mathrm{SSE} = \sum_{i=1}^{r}\sum_{j=1}^{a} e_{ij}^2$$

$$\begin{bmatrix} 144 & 101 & 130 \\ 120 & 144 & 180 \\ 176 & 211 & 141 \\ 288 & 288 & 374 \\ 144 & 72 & 302 \end{bmatrix} = \begin{bmatrix} 187.7 & 187.7 & 187.7 \\ 187.7 & 187.7 & 187.7 \\ 187.7 & 187.7 & 187.7 \\ 187.7 & 187.7 & 187.7 \\ 187.7 & 187.7 & 187.7 \end{bmatrix} + \begin{bmatrix} -13.3 & -24.5 & 37.7 \\ -13.3 & -24.5 & 37.7 \\ -13.3 & -24.5 & 37.7 \\ -13.3 & -24.5 & 37.7 \\ -13.3 & -24.5 & 37.7 \end{bmatrix}$$

$$+ \begin{bmatrix} -30.4 & -62.2 & -95.4 \\ -54.4 & -19.2 & -45.4 \\ 1.6 & 47.8 & -84.4 \\ 113.6 & 124.8 & 148.6 \\ -30.4 & -91.2 & 76.6 \end{bmatrix}$$

$$\mathrm{SSE} = (-30.4)^2 + (-54.4)^2 + \ldots + (76.6)^2 = 94{,}365.20$$

One Factor Experiments

Allocation of Variation

- The total variation can be allocated to the factor and errors.

$$SSY = SS0 + SSA + SSE$$

  - SSY (sum of squares)
  - SS0 (sum of squares of grand means),
  - SSA (sum of squares of effects)
  - SSE (sum of square errors).

$$SST = \sum_{i,j}(y_{i,j} - \bar{y}_{..})^2 = \sum_{i,j} y_{ij}^2 - ar\bar{y}_{..}^2 = SSY - SS0 = SSA + SSE$$

Well

Programa

1. CTMC - Métodos de Análise Estacionária; Textos 6
2. CTMC - Métodos de Análise Transiente; Textos 6
3. Métodos para Análise de Sensibilidade; Texto 10
4. *Desing and Analysis of Experiment*; Textos 1 e 4
5. Regressão; Texto 7 e 4
6. Uma Introdução às Séries Temporais; Texto 8
7. Gráficos de Controle Estatísticos (X, R, S, CUSUM, EWMA); Texto 4.

One Factor Experiments

- **Analysis of Variance(ANOVA)**
  - The **partitioning of variation** into an
    - explained
    - unexplained

  part is useful in practice since it can be easily presented by the analyst to the decision makers

  - To determine if a factor has a **significant effect** on the response, statisticians compare its contribution to the variation with that of the errors.

One Factor Experiments

- **Analysis of Variance(ANOVA)**
  - What is **ANOVA**?
    - the statistical procedure to analyze the significance os various factors
  - **Degree of freedom**
    - number of **independent values** required to compute the **sum of squares**

One Factor Experiments

- **Analysis of Variance(ANOVA)**
  - Result:

ANOVA Table for One-Factor Experiments

Component	Sum of Squares	Percentage of Variation	Degrees of Freedom	Mean Square	F-Computed	F-Table
y	$SSY = \sum y_{ij}^2$		ar			
$\bar{y}_{..}$	$SSO = ar\mu^2$		1			
$y - \bar{y}_{..}$	SST = SSY - SSO	100	$ar - 1$			
A	$SSA = r\sum \alpha_i^2$	$100\left(\frac{SSA}{SST}\right)$	$a - 1$	$MSA = \frac{SSA}{a-1}$	$\frac{MSA}{MSE}$	$F_{[1-\pm;\ a-1, a(r-1)]}$
e	SSE = SST - SSA	$100\left(\frac{SSE}{SST}\right)$	$a(r - 1)$	$MSE = \frac{SSE}{a(r-1)}$		

One Factor Experiments

Confidence Intervals for Effects

Parameter Estimation from One-Factor Experiments

Parameter	Estimate	Variance
μ	$\bar{y}_{..}$	s_e^2/ar
α_j	$\bar{y}_{.j} - \bar{y}_{..}$	$s_e^2(a-1)/ar$
$\mu + \alpha_j$	$\bar{y}$	s_e^2/r
$\sum_{j=1}^{a} h_j\alpha_j,$		
$\sum_{j=1}^{a} h_j = 0$	$\sum_{j=1}^{a} h_j\bar{y}_{.j}$	$\sum_{j=1}^{a} s_e^2 h_j^2/ar$
s_e^2	$\sum e_{ij}^2/[a(r-1)]$	

Two Factor Factorial Experiments

- Used when there are two parameters that are carefully controlled and varied to study their impact on the performance.
- Model used:

$$y_{ij} = \mu + \alpha_j + \beta_i + e_{ij}$$

- Observation, mean response, effect of factor A at level j, effect of factor B at level i, and error term.

Two Factor Factorial Experiments

- **The values of model parameters are computed such that the error has a zero mean. Sum of each column and each row is zero.**

$$\bar{y}_{.j} = \mu + \alpha_j + \frac{1}{b}\sum_i \beta_i + \frac{1}{b}\sum_i e_{ij}$$

$$\bar{y}_{.j} = \mu + \alpha_j$$

$$\bar{y}_{i.} = \mu + \beta_i$$

$$\bar{y}_{..} = \mu$$

Two Factor Factorial Experiments

Confidence Intervals for Effects

TABLE 21.2 Computation of Effects for the Cache Comparison Study

Workload	Two Caches	One Cache	No Cache	Row Sum	Row Mean	Row Effect
ASM	54.0	55.0	126.0	215.0	71.7	-0.5
TECO	60.0	60.0	123.0	243.0	81.0	8.8
SIEVE	43.0	43.0	120.0	206.0	68.7	-3.5
DHRYSTONE	49.0	52.0	111.0	212.0	70.7	-1.5
SORT	49.0	50.0	128.0	207.0	69.0	-3.2
Column sum	255.0	260.0	568.0	1283.0		
Column mean	51.0	52.0	113.6		72.2	
Column effect	-21.2	-20.2	41.4			

Two Factor Factorial Experiments

Estimating Experimental Errors

$$\hat{y}_{ij} = \mu + \alpha_j + \beta_i$$

$$e_{ij} = y_{ij} - \hat{y}_{ij} = y_{ij} - \mu - \alpha_j - \beta_i$$

TABLE 21.3 Error Computation for the Cache Comparison Study

Workload	Two Caches	One Cache	No Cache
ASM	3.5	3.5	-7.1
TECO	0.2	-0.8	0.6
SIEVE	-4.5	-5.5	9.9
DHRYSTONE	-0.5	1.5	-1.1
SORT	1.2	1.2	-2.4

The measured processor time is 54 milliseconds. The difference 54 - 50.5 = 3.5 is the error.

$SSE = (3.5)^2 + (0.2)^2 + \ldots + (-2.4)^2 = 2368.00$

Two Factor Factorial Experiments

Allocation of Variation

SSY = SS0 + SSA + SSB + SSE

The total variation (SST)

SST = SSY - SS0 = SSA + SSB + SSE

Example 21.3 For the cache comparison study of Example 21.1, the sums of squares are

Two Factor Factorial Experiments

Analysis of Variance

- **To statistically test the significance of a factor, we divide the sum of squares by their corresponding degrees of freedom to get mean squares.**

Two Factor Factorial Experiments

Article

Openstack Scheduler Evaluation using Design of Experiment Approach

Oleg Litvinski and Abdelouahed Gherbi
Department of Software and IT Engineering
École de technologie supérieure
{oleg.litvinski.1, abdelouahed.gherbi} @etsmtl.ca
Montreal, Canada

Two Factor Factorial Experiments

Article

TABLE I. TERMS AND DEFINITION

Name	Description				
	Details	*Min*	*Mean*	*Max*	
VRAM	Amount of RAM allocated to VM (GB)	0.5	4	8	Request Generator
VCPU	Number of CPU allocated to VM	0	2	4	
SRAM small	Amount of RAM available on Physical Node with 4GB (GB)	0	2	4	Load Generator
SRAM large	Amount of RAM available on Physical Node with 8GB (GB)	0	4	8	
SCPU small	Number of CPU available on Physical Node with 2 core	0	1	2	
SCPU large	Number of CPU available on Physical Node with 4 core	0	2	4	
RAM small	Amount of RAM available on Node with 4GB (GB)	Scheduler Observer			
RAM large	Amount of RAM available on Node with 8GB (GB)				
CPU small	Number of CPU available on Node with 2 core				
CPU large	Number of CPU available on Node with 4 core				

TABLE II. ANOVA SUMMARY FOR NUMBER OF CPU.

Name	Adjusted Model			
	F-value		*p-value*	
	CPU small	*CPU large*	*CPU small*	*CPU large*
Model	2.9702	5.0023	0.0513	0.0092
Curvature	0.5940	0.9166	0.4558	0.3589
Lack of Fit	0.4318	0.1363	0.8558	0.9890
	Unadjusted Model			
	F-value		*p-value*	
	CPU small	*CPU large*	*CPU small*	*CPU large*
Model	3.0660	5.0373	0.0427	0.0073
Lack of Fit	0.4227	0.1590	0.8677	0.9863

Two Factor Factorial Experiments

Article

TABLE III. ANOVA SUMMARY FOR AMOUNT OF MEMORY.

Name		Adjusted Model			
		F-value		*p-value*	
		RAM small	*RAM large*	*RAM small*	*RAM large*
Model	Filter	5.2254	5.2000	0.0076	0.0091
	Chance	7.1828	8.6254	0.0036	0.0013
	Simple	2.0528	7.1309	0.1497	0.0028
Curvature	Filter	6.6414	8.9948	0.0230	0.0134
	Chance	9.1041	21.9409	0.0145	0.0009
	Simple	0.0303	16.1281	0.8641	0.0025
Lack of Fit	Filter	1.2784	0.7457	0.4695	0.6639
	Chance	1.7835	0.3503	0.3397	0.8857
	Simple	0.7722	11.0901	0.6789	0.0369
		Unadjusted Model			
		F-value		*p-value*	
		RAM small	*RAM large*	*RAM small*	*RAM large*
Model	Filter	3.7245	3.0113	0.0236	0.0469
	Chance	3.9675	2.9704	0.0213	0.0489
	Simple	2.1852	3.0021	0.1296	0.0473
Lack of Fit	Filter	1.8953	1.5767	0.3276	0.3865
	Chance	3.5086	1.8018	0.1651	0.3408
	Simple	0.7147	25.9591	0.7126	0.0109

TABLE V. IMPORTANCE OF FACTORS AND ITS COMBINATION IN SCHEDULERS CHOICE OF NODE

CPU small	*CPU large*	*RAM small*			*RAM large*		
All Scheduler		Filter	Chance	Simple	Filter	Chance	Simple
A	A	A	A	A	A	A	A
B	B	B	B	B	B	B	B
C	C	E	C	BF	D	D	D
E	E	AB	E		F	E	F
AB	F	AE	F		AB	F	AB
AE	AC		AB		AF	AE	AD
	BF		AF		BD	BD	BD
			BF		BF	BF	
			ABF				

Keys: A –VRAM, B- VCPU, C – SCPU small, D – SCPU large, E – SRAM small, F –SRAM large.

2ˆk Factorial Designs

- Used to determine the effect of k factors (with two alternatives or levels).
- Easy to analyse and helps in sorting factors in the order of impact.
- With a large number of factors and levels may not be the best use of available effort
- The first step should be to reduce the number of factors and choose those that have significant impact.

2ˆk Factorial Designs

Example

TABLE 17.1 Performance in MIPS

Cache Size (kbytes)	Memory Size 4 Mbytes	Memory Size 16 Mbytes
1	15	45
2	25	75

Nonlinear Regression Model → $y = q_0 + q_A x_A + q_B x_B + q_{AB} x_A x_B$

2ˆk Factorial Designs

Example

TABLE 17.2 Analysis of a 2^2 Design

Experiment	A	B	y
1	–1	–1	y^1
2	1	–1	y^2
3	–1	1	y^3
4	1	1	y^4

2^k Factorial Designs

Other Way: General

$$y_1 = q_0 - q_A - q_B + q_{AB}$$

$$y_2 = q_0 + q_A - q_B - q_{AB}$$

$$y_3 = q_0 - q_A + q_B - q_{AB}$$

$$y_4 = q_0 + q_A + q_B + q_{AB}$$

$$q_0 = \frac{1}{4}(y_1 + y_2 + y_3 + y_4)$$

$$q_A = \frac{1}{4}(-y_1 + y_2 - y_3 + y_4)$$

$$q_B = \frac{1}{4}(-y_1 - y_2 + y_3 + y_4)$$

$$q_{AB} = \frac{1}{4}(y_1 - y_2 - y_3 + y_4)$$

2^k Factorial Designs

Allocation of Variation

- The importance of a factor is measured by the proportion of the total variation.

$$\text{Sample variance of } y = s_y^2 = \frac{\sum_{i=1}^{2^2}(y_i - \bar{y})^2}{2^2 - 1}$$

- $\bar{y}$ denotes the mean of responses from all four experiments.
- Sums of Squares Total (SST):

$$\text{Total variation of } y = \text{SST} = \sum_{i=1}^{2^2}(y_i - \bar{y})^2$$

2ˆk Factorial Designs

Allocation of Variation

- **SST also can be represented by:**

$$SST = 2^2 q_A^2 + 2^2 q_B^2 + 2^2 q_{AB}^2$$

- **Effect of A, B, and interaction AB.**

$$SST = SSA + SSB + SSAB$$

$$\text{Fraction of variation explained by } A = \frac{SSA}{SST}$$

- **This fraction provides the importance of the factor A.**

2ˆk Factorial Designs

Going Back To the Example

2ˆk Factorial Designs

Article

Using Statistical Design of Experiments for Analyzing Mobile Ad Hoc Networks*

Michael W. Totaro
The Center for Advanced Computer Studies
University of Louisiana at Lafayette
Lafayette, LA 70504-4330
miket@cacs.louisiana.edu

Dmitri D. Perkins
The Center for Advanced Computer Studies
University of Louisiana at Lafayette
Lafayette, LA 70504-4330
perkins@cacs.louisiana.edu

2^k Factorial Designs

Article

Table 1: Experimental Factors

Label	Factor	Level 1 (−)	Level 2 (+)
1	Avg. neighbors	7 (strongly-connected)	3 (weakly-connected)
2	Avg. node speed	5 m/sec (1-10 m/sec range)	30 m/sec (25-35 m/sec range)
3	Traffic load	10% of no. of nodes	20% of no. of nodes
4	No. of nodes	100	500
5	MAC layer	802.11b w/ RTS	802.11b w/out RTS

2^k Factorial Designs

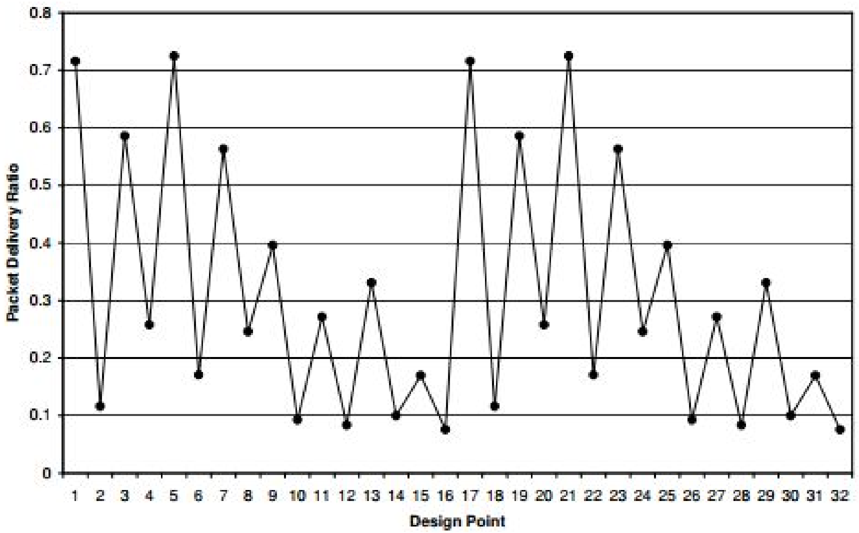
Article

Table 2: Design matrix for 2^5 factorial design

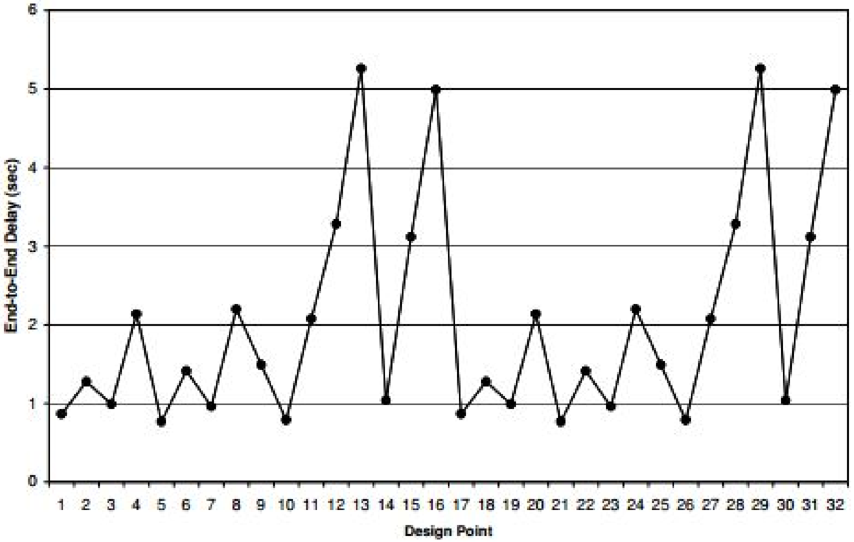
		Factors					Performance Metrics	
		1	2	3	4	5		
Design Points	Factor Level	Avg. No. of Neighbors	Avg. Node Speed (m/s)	Traffic Load	Number of Nodes	MAC Layer	Packet Delivery Ratio	End-to-End Delay (secs)
	(−)	7	5	10% of Number of Nodes	100	802.11b w/ RTS		
	(+)	3	30	20% of Number of Nodes	500	802.11b w/out RTS		
1		(−) 7	(−) 5	(−) 10	(−) 100	(−) 802.11 b w/RTS	0.71568	0.86571101
2		(+) 3	(−) 5	(−) 10	(−) 100	(−) 802.11 b w/RTS	0.11592	1.27659734
3		(−) 7	(+) 30	(−) 10	(−) 100	(−) 802.11 b w/RTS	0.58568	0.9923993
4		(+) 3	(+) 30	(−) 10	(−) 100	(−) 802.11 b w/RTS	0.25776	2.13651797
5		(−) 7	(−) 5	(+) 20	(−) 100	(−) 802.11 b w/RTS	0.72484	0.76839629
6		(+) 3	(−) 5	(+) 20	(−) 100	(−) 802.11 b w/RTS	0.17076	1.41365995
7		(−) 7	(+) 30	(+) 20	(−) 100	(−) 802.11 b w/RTS	0.563	0.96332324
8		(+) 3	(+) 30	(+) 20	(−) 100	(−) 802.11 b w/RTS	0.24584	2.19733746
9		(−) 7	(−) 5	(−) 50	(+) 500	(−) 802.11 b w/RTS	0.395968	1.49277102
10		(+) 3	(−) 5	(−) 50	(+) 500	(−) 802.11 b w/RTS	0.092656	0.78984261
11		(−) 7	(+) 30	(−) 50	(+) 500	(−) 802.11 b w/RTS	0.271504	2.07584805
12		(+) 3	(+) 30	(−) 50	(+) 500	(−) 802.11 b w/RTS	0.08344	3.28247314
13		(−) 7	(−) 5	(+) 100	(+) 500	(−) 802.11 b w/RTS	0.330824	5.25921359
14		(+) 3	(−) 5	(+) 100	(+) 500	(−) 802.11 b w/RTS	0.099736	1.04019082
15		(−) 7	(+) 30	(+) 100	(+) 500	(−) 802.11 b w/RTS	0.16395	3.11871308
16		(+) 3	(+) 30	(+) 100	(+) 500	(−) 802.11 b w/RTS	0.07568	4.98781013
17		(−) 7	(−) 5	(−) 10	(−) 100	(+) 802.11 b w/out RTS	0.71568	0.865711017
18		(+) 3	(−) 5	(−) 10	(−) 100	(+) 802.11 b w/out RTS	0.11592	1.27659734
19		(−) 7	(+) 30	(−) 10	(−) 100	(+) 802.11 b w/out RTS	0.58568	0.9923993
20		(+) 3	(+) 30	(−) 10	(−) 100	(+) 802.11 b w/out RTS	0.25776	2.13651797
21		(−) 7	(−) 5	(+) 20	(−) 100	(+) 802.11 b w/out RTS	0.72484	0.76839629
22		(+) 3	(−) 5	(+) 20	(−) 100	(+) 802.11 b w/out RTS	0.17076	1.41365995
23		(−) 7	(+) 30	(+) 20	(−) 100	(+) 802.11 b w/out RTS	0.563	0.96332324
24		(+) 3	(+) 30	(+) 20	(−) 100	(+) 802.11 b w/out RTS	0.24584	2.19733746
25		(−) 7	(−) 5	(−) 50	(+) 500	(+) 802.11 b w/out RTS	0.395968	1.49277102
26		(+) 3	(−) 5	(−) 50	(+) 500	(+) 802.11 b w/out RTS	0.092656	0.78984261
27		(−) 7	(+) 30	(−) 50	(+) 500	(+) 802.11 b w/out RTS	0.271504	2.07584805
28		(+) 3	(+) 30	(−) 50	(+) 500	(+) 802.11 b w/out RTS	0.08344	3.28247314
29		(−) 7	(−) 5	(+) 100	(+) 500	(+) 802.11 b w/out RTS	0.330824	5.25921359
30		(+) 3	(−) 5	(+) 100	(+) 500	(+) 802.11 b w/out RTS	0.099736	1.04019082
31		(−) 7	(+) 30	(+) 100	(+) 500	(+) 802.11 b w/out RTS	0.16952	3.11871308
32		(+) 3	(+) 30	(+) 100	(+) 500	(+) 802.11 b w/out RTS	0.07568	4.98781013

2ˆk Factorial Designs

Article



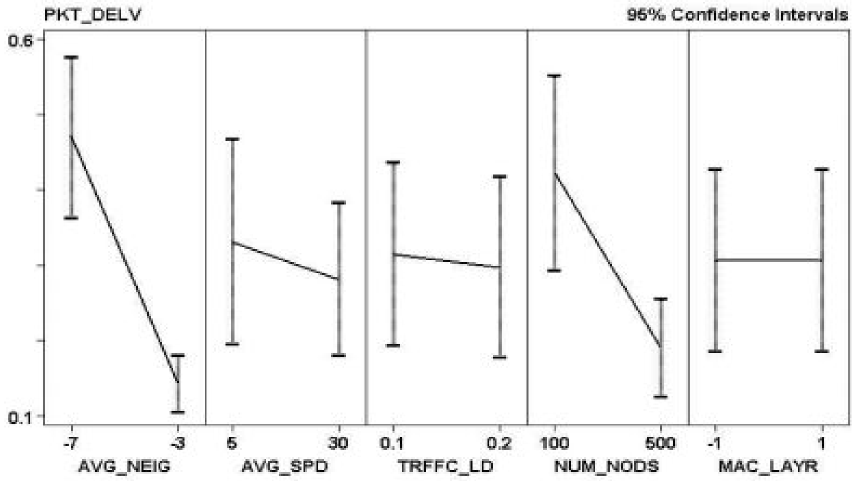
(a) Packet Delivery Ratio



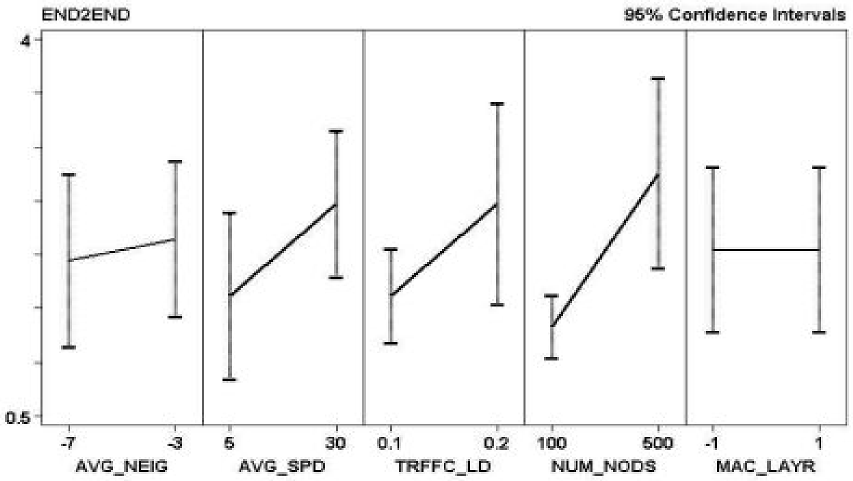
(b) End-to-end Delay

2ˆk Factorial Designs

Article



(a) Main effects - Packet delivery ratio



(b) Main effects - End-to-end delay

2ˆkr Factorial Designs

- If each of the 2^k experiments is repeated r times.
- Makes possible, estimate experimental errors through repetitions.

$$y = q_0 + q_A x_A + q_B x_B + q_{AB} x_A x_B + e$$

2ˆkr Factorial Designs

Difference

TABLE 18.1 Analysis of a $2^2 3$ Design

I	A	B	AB	y	Mean $\bar{y}$
1	–1	–1	1	(15, 18, 12)	15
1	1	–1	–1	(45, 48, 51)	48
1	–1	1	–1	(25, 28, 19)	24
1	1	1	1	(75, 75, 81)	77
164	86	38	20		Total
41	21.5	9.5	5		Total/4

2ˆkr Factorial Designs

Estimation of Experimental Errors

TABLE 18.2 Computation of Errors in Example 18.2

	Effect				Estimated Response,	Measured Responses			Errors		
	I	*A*	*B*	*AB*							
i	41	21.5	9.5	5	$\hat{y}_i$	y_{i1}	y_{i2}	y_{i3}	e_{i1}	e_{i2}	e_{i3}
1	1	–1	–1	1	15	15	18	12	0	3	–3
2	1	1	–1	–1	48	45	48	51	–3	0	3
3	1	–1	1	–1	24	25	28	19	1	4	–5
4	1	1	1	1	77	75	75	81	–2	–2	4

2^kr Factorial Designs

Allocation of Variation

- **SSE is the variation attributed to the experimental errors.**
- **Designate the variance of each term.**

$$\sum_{i,j}(y_{ij}-\bar{y}_{..})^2 = 2^2 r q_A^2 + 2^2 r q_B^2 + 2^2 r q_{AB}^2 + \sum_{i,j} e_{ij}^2$$

$$\text{SST} = \text{SSA} + \text{SSB} + \text{SSAB} + \text{SSE}$$

2^kr Factorial Designs

Confidence Intervals for Effect

- **The confidence intervals for the effects can be computed if the variance of the sample estimates are known.**

Standard Deviation of Errors

$$s_e^2 = \frac{\mathrm{SSE}}{2^2(r-1)} \longrightarrow s_e = \sqrt{\frac{\mathrm{SSE}}{2^2(r-1)}}$$

Standard Deviation of effects

$$s_{q_0} = s_{q_A} = s_{q_B} = s_{q_{AB}} = \frac{s_e}{\sqrt{2^2 r}}$$

Confidence Interval For the Effects

$$q_i \mp t_{[1-\alpha/2;2^2(r-1)]} s_{q_i}$$

2ˆkr Factorial Designs

Article

APPLICATION OF DESIGN OF EXPERIMENTS ON THE SIMULATION OF A PROCESS IN AN AUTOMOTIVE INDUSTRY

José Arnaldo Barra Montevechi

Avenida BPS, 1303 – Caixa Postal: 50
Instituto de Engenharia de Produção e Gestão
Universidade Federal de Itajubá
Itajubá, MG, 37500-903, BRAZIL

Alexandre Ferreira de Pinho
Fabiano Leal
Fernando Augusto Silva Marins

Av. Dr. Ariberto Pereira da Cunha, 333
Universidade Estadual Paulista
Guaratiguetá, SP, 12.516-410, BRAZIL

2ˆkr Factorial Designs

Article

Table 3: Experimental conditions

Factors	Level +	Level -
A: Buffer Capacity (units)	60	100
B: Number of tables (units)	4	8
C: Pre-plug operating time (min/part)	0,6	0,9
D: Table time of operation (min/part)	N(6,35 ; 0,66)	N(7,35 ; 0,66)

Table 4: Experimental matrix

	A	B	C	D	r1	r2	r3	Média	S
1	-	-	-	-	303	299	300	300,67	4,33
2	+	-	-	-	304	298	300	300,67	9,33
3	-	+	-	-	304	298	299	300,33	10,33
4	+	+	-	-	304	298	299	300,33	10,33
5	-	-	+	-	301	297	299	299,00	4,00
6	+	-	+	-	301	297	298	298,67	4,33
7	-	+	+	-	511	512	511	511,33	0,33
8	+	+	+	-	538	538	538	538,00	0,00
9	-	-	-	+	261	259	259	259,67	1,33
10	+	-	-	+	261	261	260	260,67	0,33
11	-	+	-	+	435	436	434	435,00	1,00
12	+	+	-	+	514	509	518	513,67	20,33
13	-	-	+	+	261	260	258	259,67	2,33
14	+	-	+	+	261	260	258	259,67	2,33
15	-	+	+	+	488	487	491	488,67	4,33
16	+	+	+	+	510	507	515	510,67	16,33
							Total	5836,67	91,33

2ˆkr Factorial Designs

Article

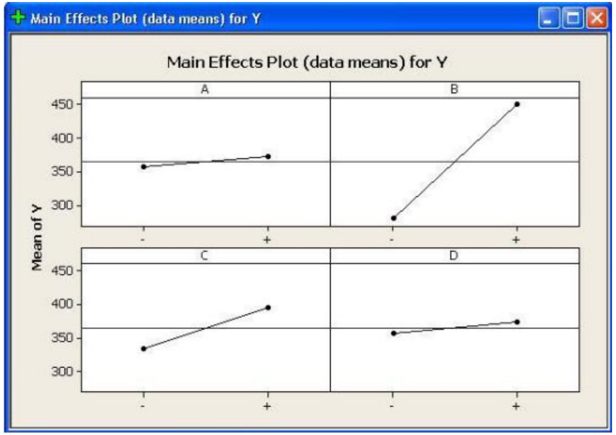


Figure 4: Graph of the main effects

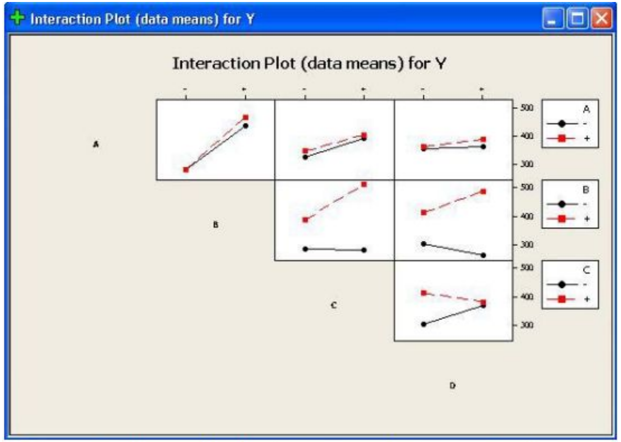


Figure 6: Second order interaction between the factors

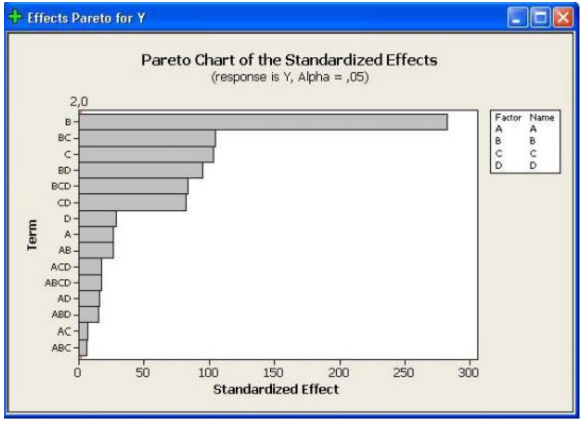


Figure 5: Pareto chart of the standardized effects

$2^{(k-p)}$ Fractional Factorial Designs

- **Aiming at:**
  - reducing the number of replications
  - Nevertheless:
    - it is possible that some combinations of factors were not possible
  - 2^{k-1}: half replication
  - 2^{k-2}: one-quarter replication

Practical Lesson

Practical Lesson

Exercise 1

Factor	Level –1	Level 1
Memory size, A	4 Mbytes	16 Mbytes
Cache size, B	1 kbyte	2 kbytes
Number of processors, C	1	2

Practical Lesson

Exercise 1

Cache Size (kbytes)	4 Mbytes		16 Mbytes	
	One Processor	Two Processor	One Processor	Two Processor
1	14	46	22	58
2	10	50	34	86

Practical Lesson

Exercise 1

I	*A*	*B*	*C*	*AB*	*AC*	*BC*	*ABC*	y
1	–1	–1	–1	1	1	1	–1	14
1	1	–1	–1	–1	–1	1	1	22
1	–1	1	–1	–1	1	–1	1	10
1	1	1	–1	1	–1	–1	–1	34
1	–1	–1	1	1	–1	–1	1	46
1	1	–1	1	–1	1	–1	–1	58
1	–1	1	1	–1	–1	1	–1	50
1	1	1	1	1	1	1	1	86
320	80	40	160	40	16	24	9	Total
40	10	5	20	5	2	3	1	Total/8

800/4512 (18%), 200/4512 (4%), 3200/4512 (71%), 200/4512 (4%), 32/4512 (1%), 72/4512 (2%), and 8/4512 (0%), respectively.

References

- Jain, Raj. *The art of computer systems performance analysis*. John Wiley & Sons, 2008.
- Montgomery, Douglas C., and George C. Runger. *Applied statistics and probability for engineers*. John Wiley & Sons, 2010.
- Totaro, Michael W., and Dmitri D. Perkins. "Using statistical design of experiments for analyzing mobile ad hoc networks." *Proceedings of the 8th ACM international symposium on Modeling, analysis and simulation of wireless and mobile systems*. ACM, 2005.
- Montevechi, José Arnaldo Barra, et al. "Application of design of experiments on the simulation of a process in an automotive industry." *Proceedings of the 39th conference on Winter simulation: 40 years! The best is yet to come*. IEEE Press, 2007.
- Litvinski, Oleg, and Abdelouahed Gherbi. "Openstack scheduler evaluation using design of experiment approach." *Object/Component/Service-Oriented Real-Time Distributed Computing (ISORC), 2013 IEEE 16th International Symposium on*. IEEE, 2013.